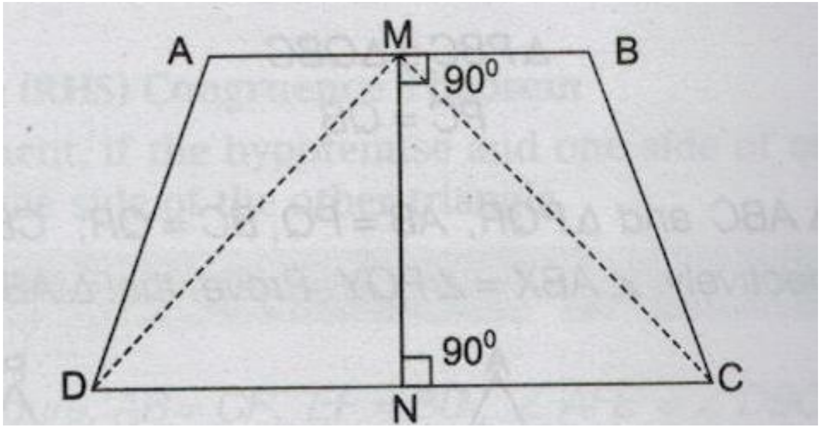
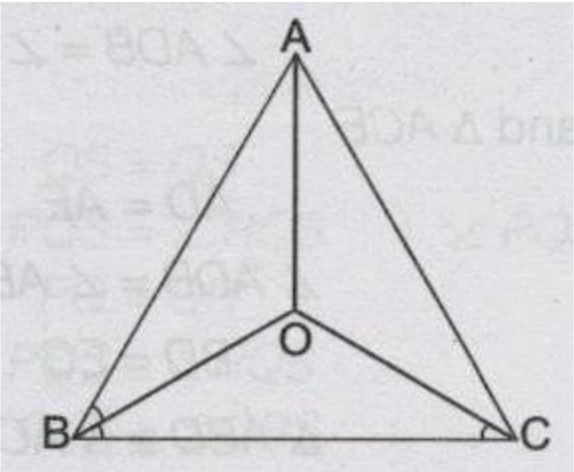
Olympiad - Level 2 training

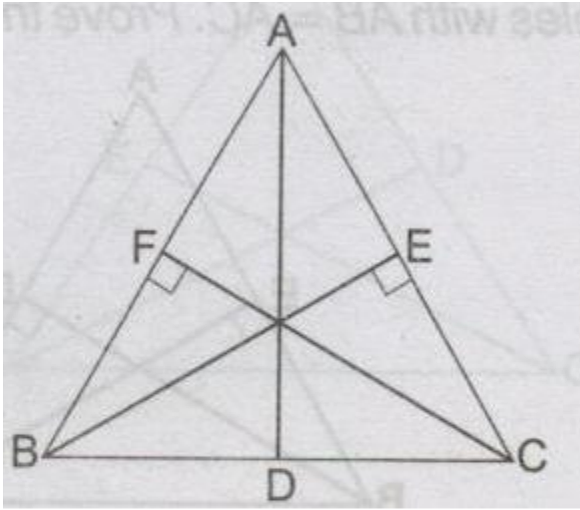
1) The line segment joining the midpoint M and N of opposite sides AB and DC of quadrilateral ABCD is perpendicular to both these sides. Prove that the other sides of the quadrilateral are equal.



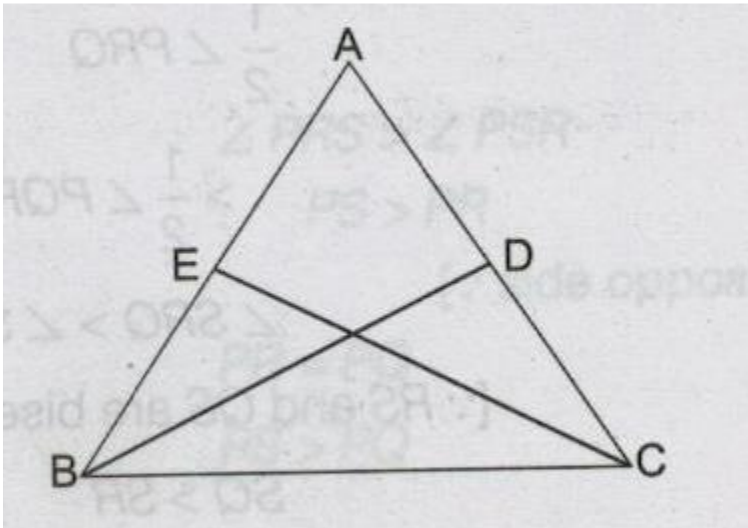
2) In triangle ABC, AB =BC . Bisectors of angles B and C intersect at point O. Prove that BO= CO and the ray AO is bisector of angle BAC.



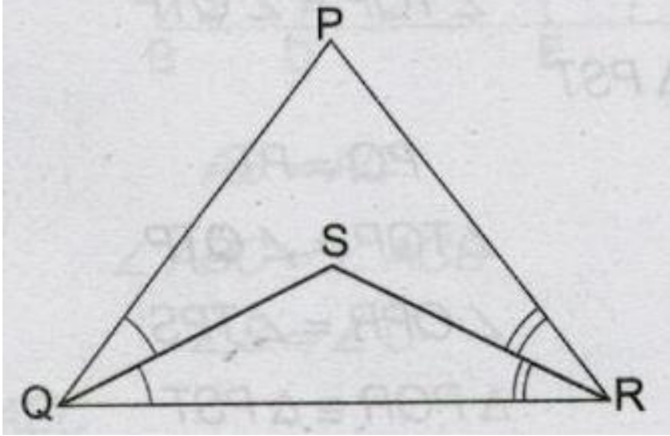
3) The altitudes of triangle ABC, AD,BE and CF are equal. Prove that triangle ABIS is an equilateral triangle.



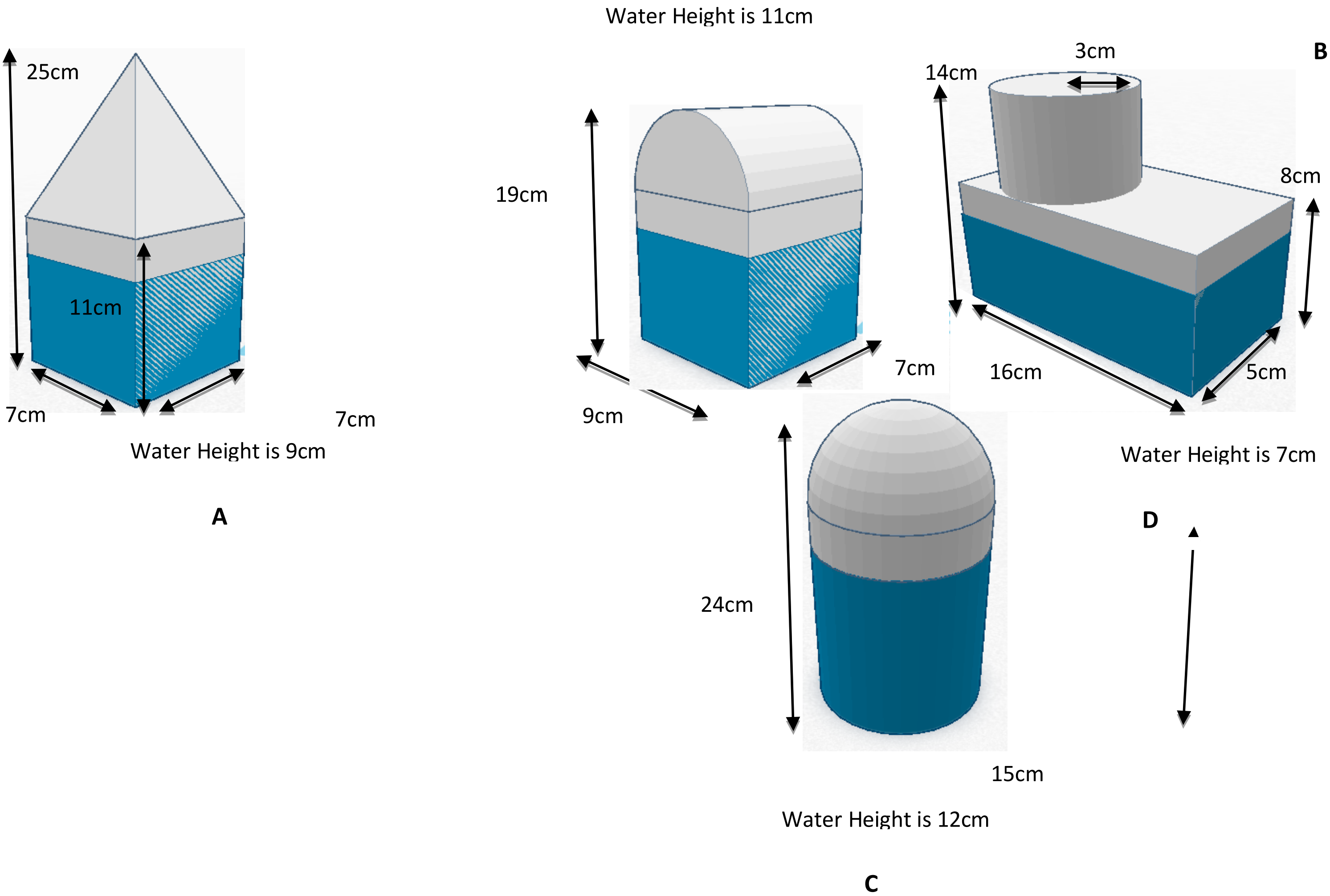
4) Triangle ABC is an isosceles triangle with AB = AC. BD and CE are two medians of triangle. Prove that BD= CE.



5) In figure PQ > PR. QS and RS are the bisectors of angle Q and angle R respectively. Prove that SQ > SR.



6) For each container below, what percentage is filled with water? Place the containers in descending order of proportion filled.



7)

Simplify the expression

$$\sqrt{\sin^4 x + 4\cos^2 x} - \sqrt{\cos^4 x + 4\sin^2 x}.$$

8)

Let a, b, c be real numbers, all different from -1 and 1, such that $a+b+c = abc$. Prove that

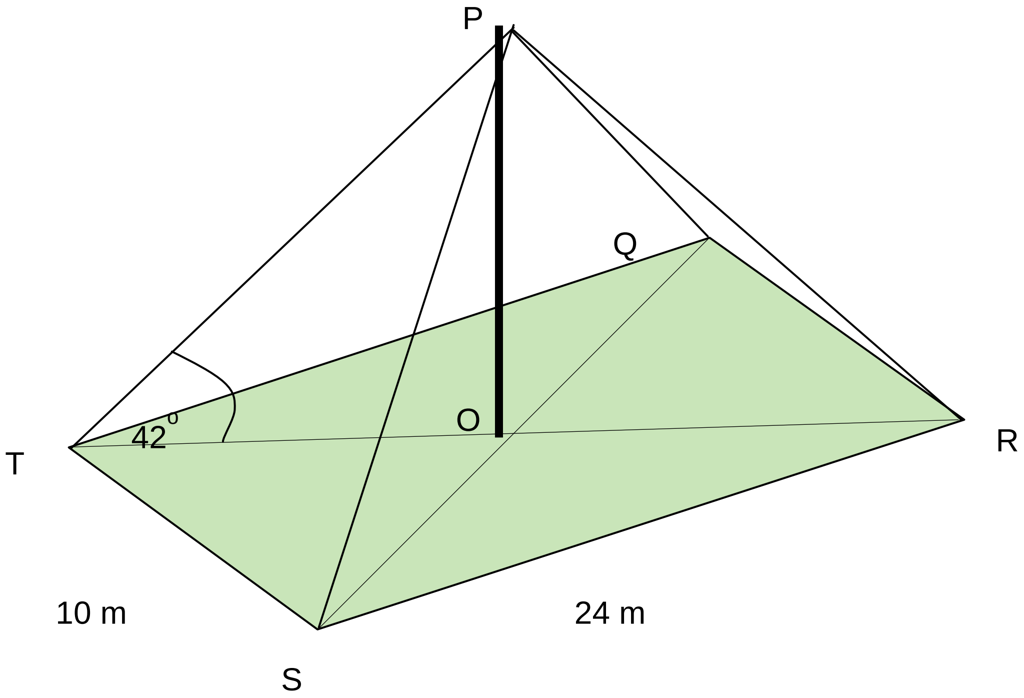
$$\frac{a}{1-a^2} + \frac{b}{1-b^2} + \frac{c}{1-c^2} = \frac{4abc}{(1-a^2)(1-b^2)(1-c^2)}.$$

9)

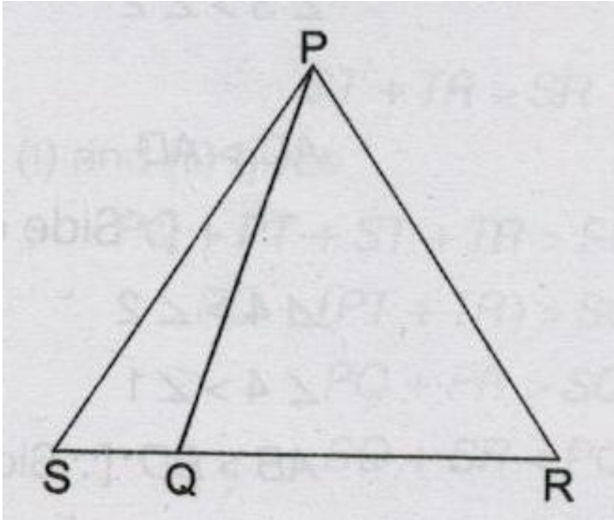
In triangle ABC, show that

$$\sin\frac{A}{2} \le \frac{a}{b+c}.$$

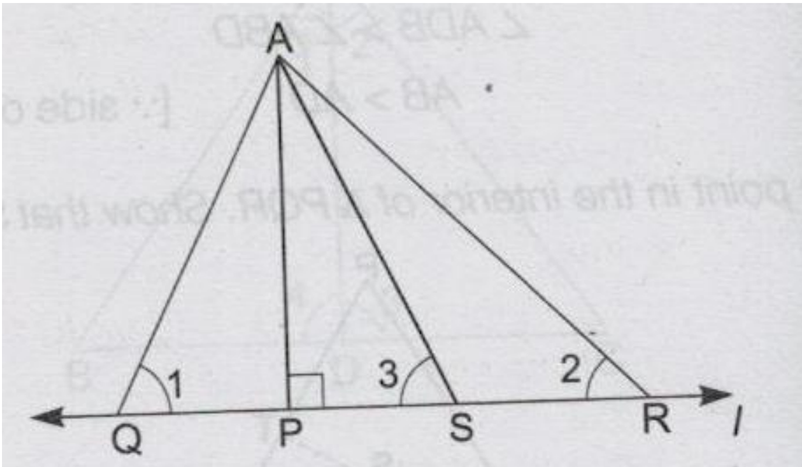
10) A vertical flag pole OP stands in the centre of a horizontal field QRST. Using the information given in the diagram, calculate the height of the flag pole.



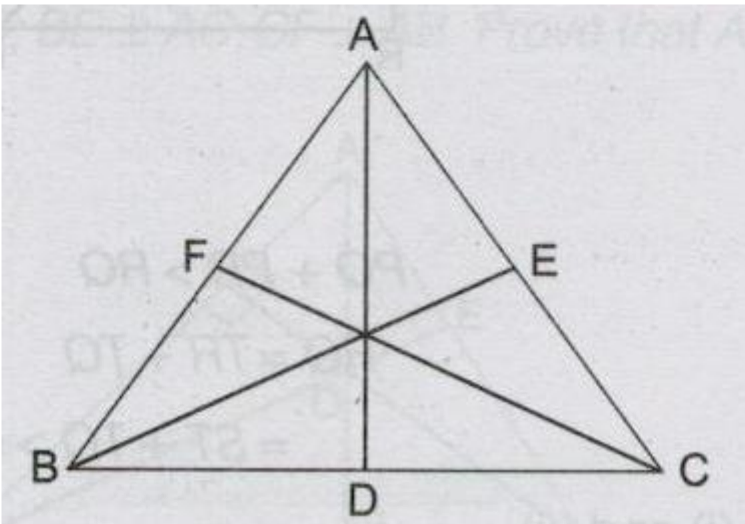
11) Q is a point on side RS of Triangle PSR such that PQ= PR, Show that PS >PQ.



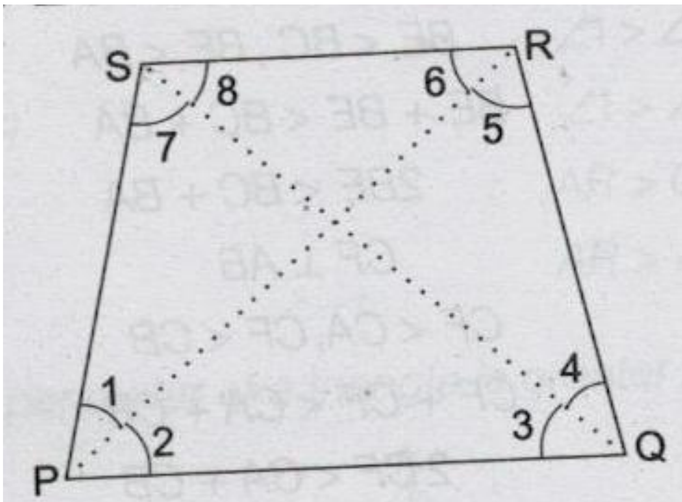
12) In the figure AP is perpendicular to L i.e. AP is the shortest line segment that can be drawn from A to the line L. If PR > PQ. Show that AR > AQ.



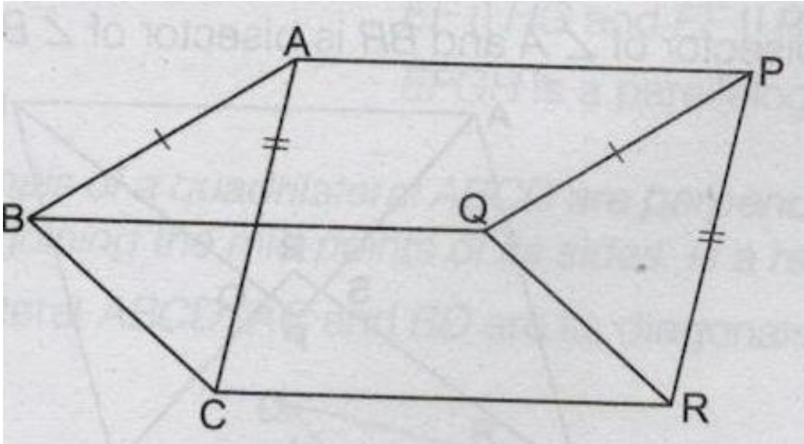
13) Show that the sum of the three altitudes of a triangle is less than the sum of the three sides of a triangle



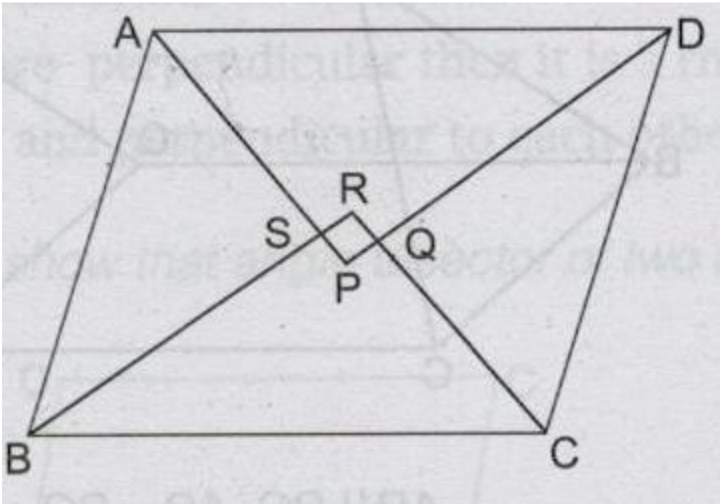
14) PQRS is a quadrilateral. PQ is the longest side. RS is the shortest side, prove that angle R > Angle P and angle S > angle Q.



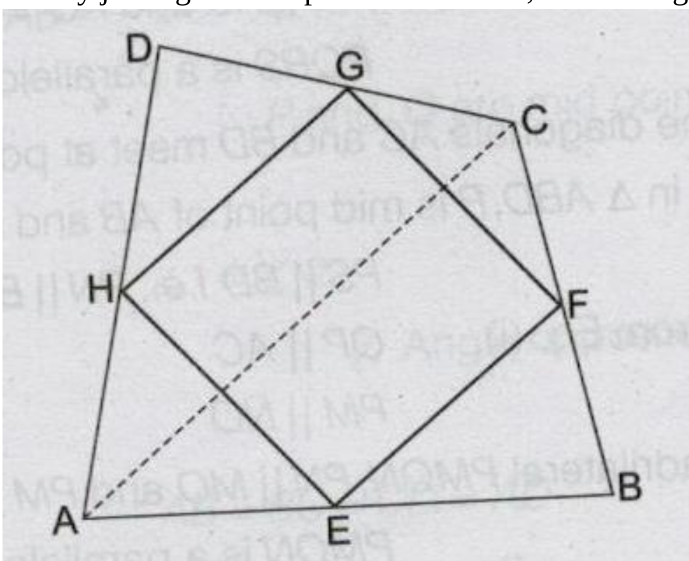
15) AB || PQ, AB= PQ, AC || PR, AC= PR. Prove that BC || QR and BC = QR.



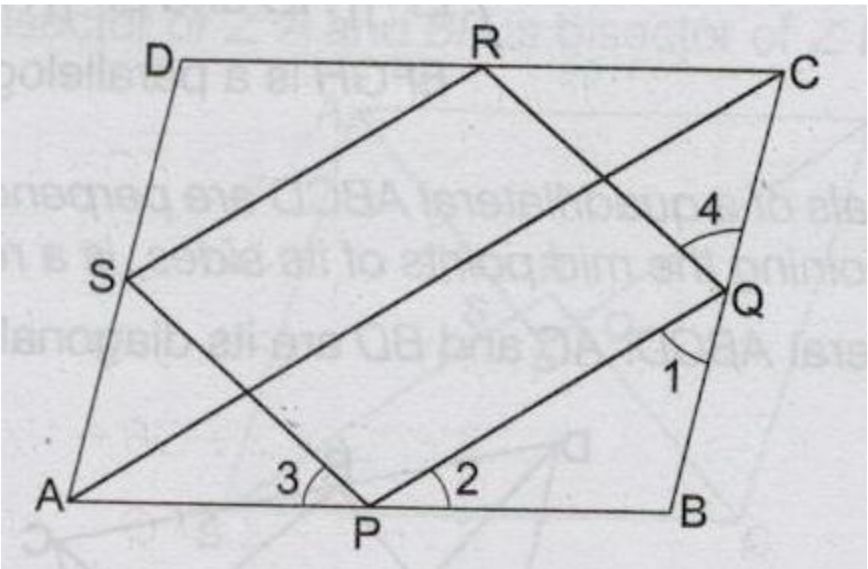
16) Prove that the angle bisector of a parallelogram forms a rectangle.



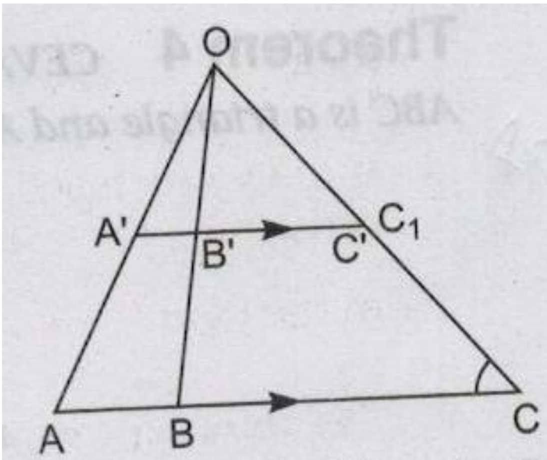
17) The diagonals of a quadrilateral ABCD are perpendicular. Show that the quadrilateral formed by joining the mid points of its sides, is a rectangle.



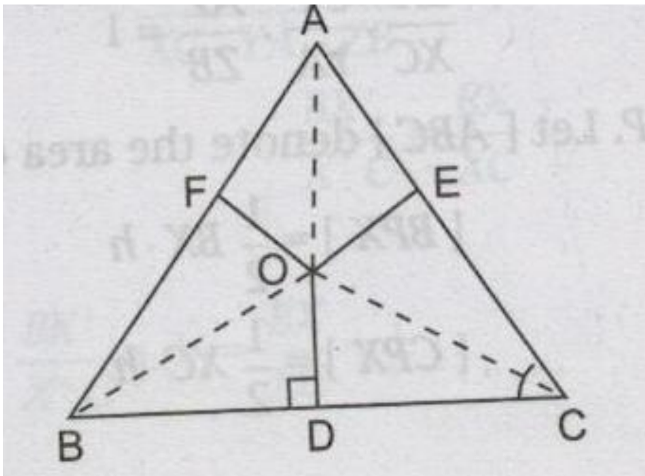
18) ABCD is a rhombus. P, Q, R, S are midpoints of AB, BC, CD, DA respectively. Prove that PQRS is a rectangle.



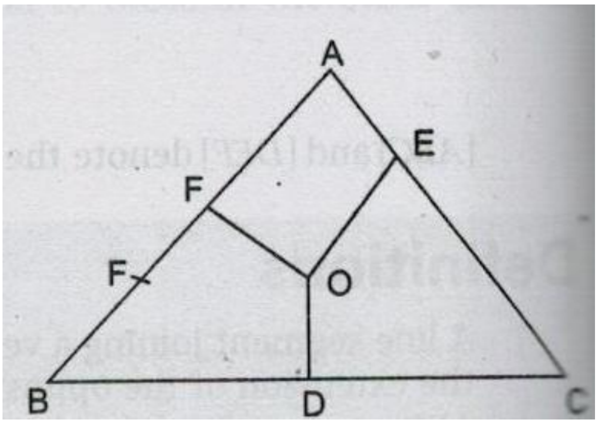
19) If A, B, C and A', B', C' are points on two parallel lines such that AB / A'B' = BC B'C', Then AA', BB', CC' are concurrent, if they are not parallel.



20) From a point O; OD, OE, OF are drawn perpendicular to the sides BC, CA and AB respectively of a triangle ABC, then
$BD^2 - DC^2 + CE^2 - EA^2 + AF^2 - FB^2 = 0$



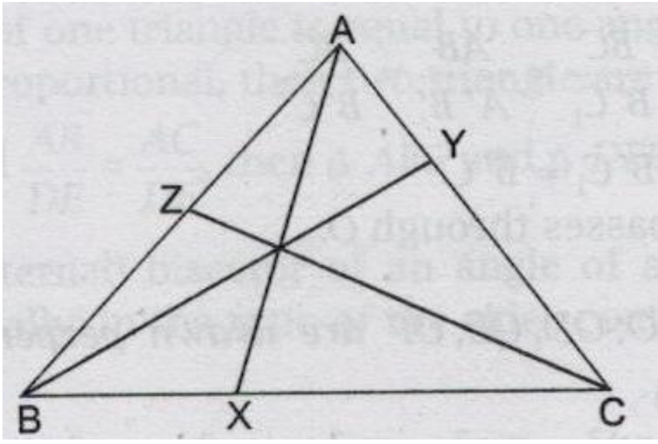
21) If D, E, F be points on sides BC, CA, AB of a triangle ABC such that $BD^2 - DC^2 + CE^2 - EA^2 + AF^2 - FB^2 = 0$ then perpendiculars at D, E, F to the respective sides are concurrent.



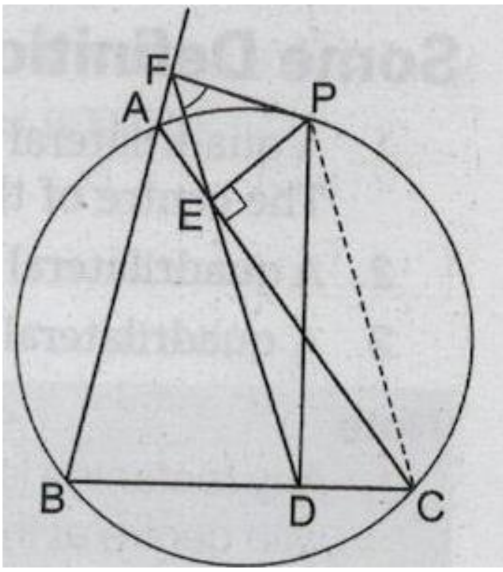
22) Cevas's Theorem – A line segment joining a vertex of a triangle to any point on the opposite side (the point may be on the extension of the opposite side also) is called a Cevian.

ABC is a triangle and AX, BY and CZ are three concurrent cevians. Then,

$$\frac{BX}{XC} \cdot \frac{CY}{YA} \cdot \frac{AZ}{ZB} = 1$$

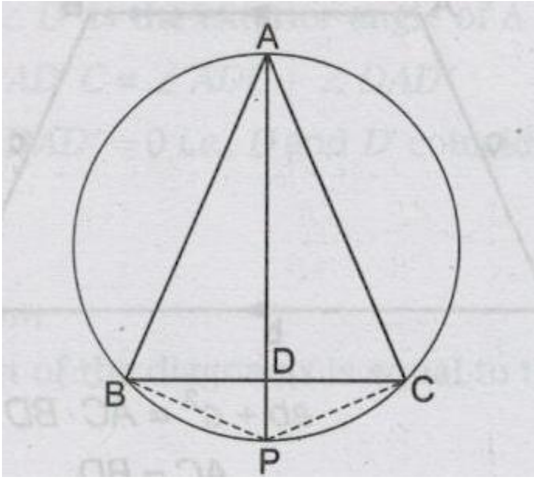


23) SIMPSON'S LINE - Prove that the feet of perpendiculars drawn from a point on the circumcircle of a triangle on the sides are collinear.

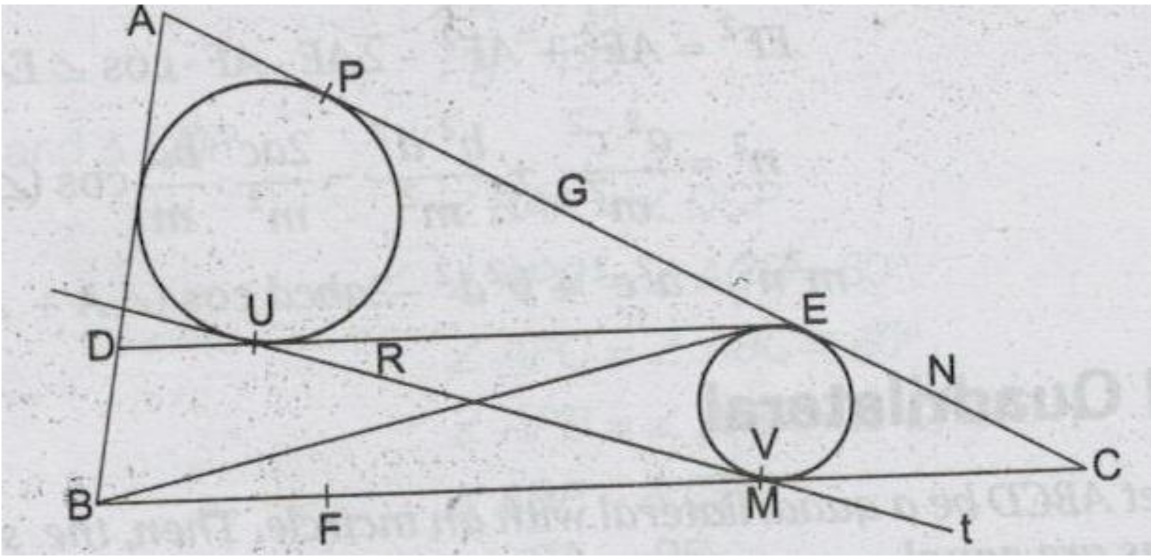


24) A line drawn from vertex A of an equilateral triangle ABC meets BC and D and the circumcircle at P. Prove that

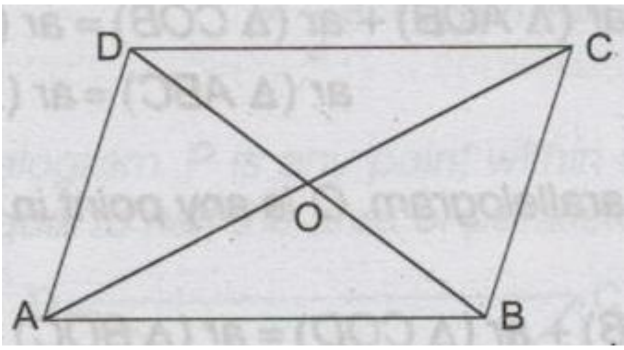
(a) PA = PB + PC (b) $\frac{1}{PD} = \frac{1}{PB} + \frac{1}{PC}$



25) Let the incircle of triangle ABC touch AB at D and let E be a point on the side AC. Prove that the incircles of triangle ADE, triangle BCE and triangle BDE have common tangents.



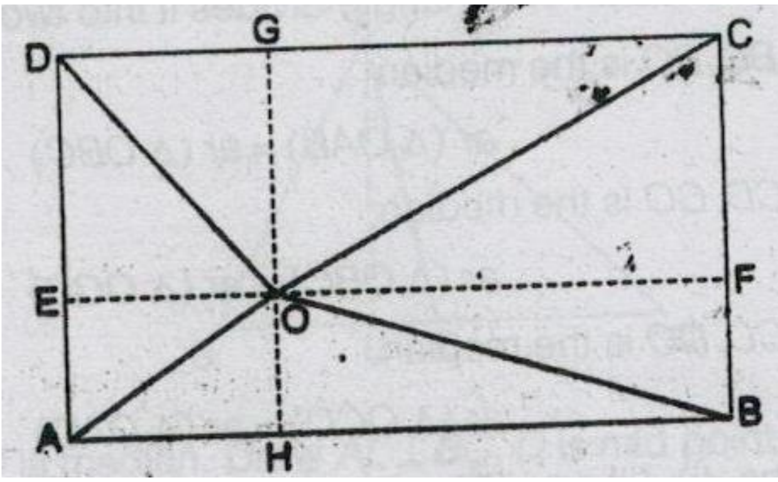
26) Show that the diagonals of a parallelogram divide it into four triangles of equal area.



27) ABCD is a parallelogram, O is any point in its interior. Prove that

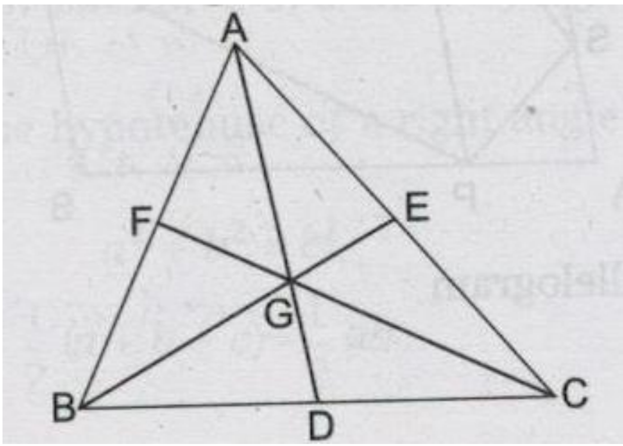
(a) Area (triangle AOB) + Area (triangle COD) = Area (triangle BOC) + Area (triangle AOD)

(b) Area (triangle AOB) + Area (triangle COD) = ½ Area (|| gm ABCD)

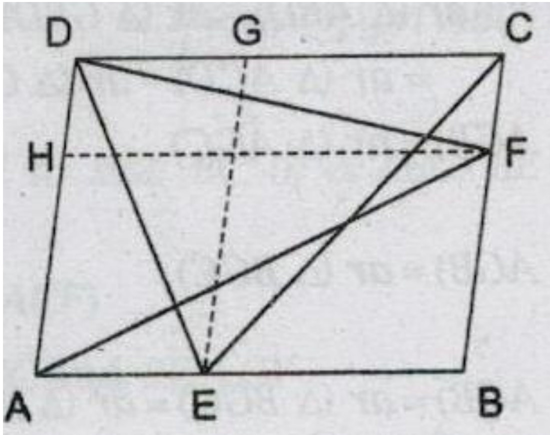


28) If the medians of a triangle ABC intersect at G. Show that

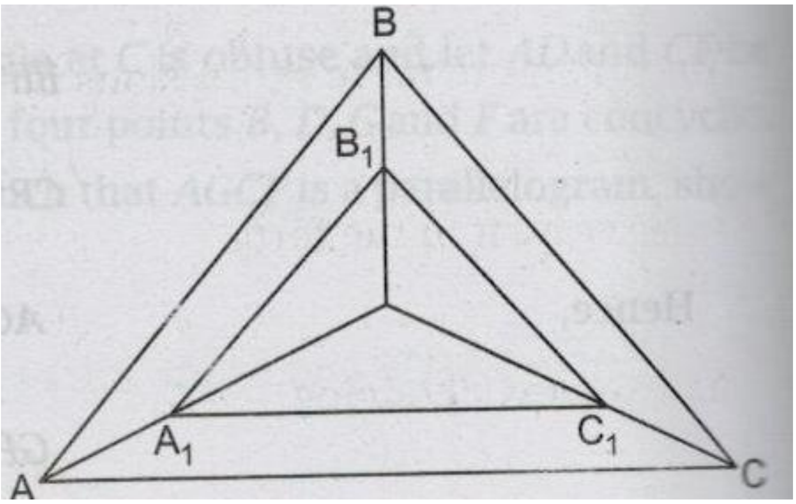
Area (triangle AGB) = Area (triangle AGC) = Area (triangle BGC) =1/3*Area (triangle ABC)



29) In a ||gm ABCD, E and F are any two points on side AB and BC respectively. Show that arear (Triangle ADF) = area (Triangle DCE)



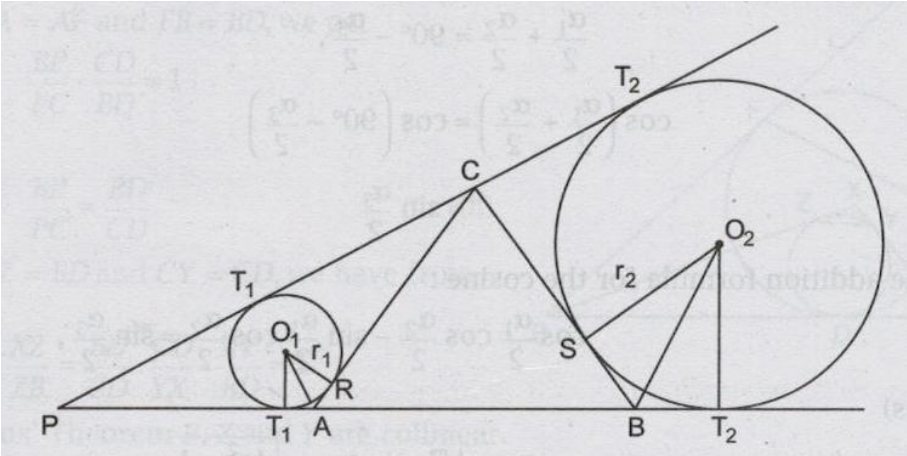
30) Let A, B and C be non-collinear points, prove that there is a unique point X in the plane of ABC such that
$XA^2 + XB^2 + AB^2 = XB^2 + XC^2 + BC^2 = XC^2 + XA^2 + CA^2$



31) A hexagon is inscribed in a circle with radius r. Two of its sides have length 1, two have length 2 and the last two have length 3, prove that r is a root of the equation.

$$2r^3 – 7r -3 = 0$$

32) Let ABC be equilateral. On side AB produced, we choose a point P such that A lies between P and B. We now denote a as the length of sides of triangle ABC; r1 as the radius of incircle of triangle PAC; and r2 as the exradius of triangle PBC with respect to side BC. Determine the sum r1 + r2 as a function of a alone.



33) Let T be an acute triangle. Inscribe a pair R, S of rectangles in T as shown: Let A(X) denote the area of polygon X. Find the maximum value, or show that no maximum exists, of (A (R) + A (S)) / A(T) where T ranges over all triangles and R, S over all rectangles as below.